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Introduction to wireless communication and OFDM

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This lecture is a brief on OFDM and its origins. The lecture will review basic digital communications principles and basic constellation. Also reviewed is multi-carrier modulation and several explanations for OFDM are given. A part of this lecture is taken from the works of Doron Ezri[].

I. Digital communication

In the field of digital communication bits are assigned to symbols, which are varying phase and amplitude, and using pulse shape filter for forming the base-band signal. The amount of symbols may vary, there is BPSK (AKA 2PAM) for just two symbols, QPSK (AKA 4 QAM) for four complex symbols, 16 QAM for 16 different symbols etc.

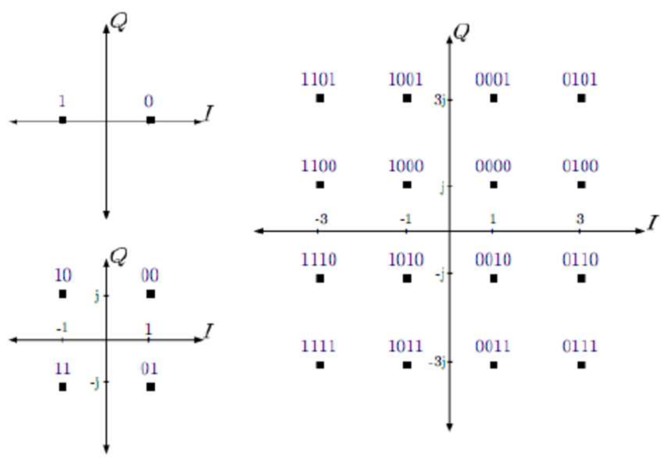

Fig. 1. figure of 2/4/16 QAM

After converting string of bit to string of symbols, it then combined with pulse shape filter to form a baseband signal or code line. Baseband is whats known as information signal and it is modulated and sent. Pulse shape filter can use one of many shapes i.g. rectangular, Gaussian etc.

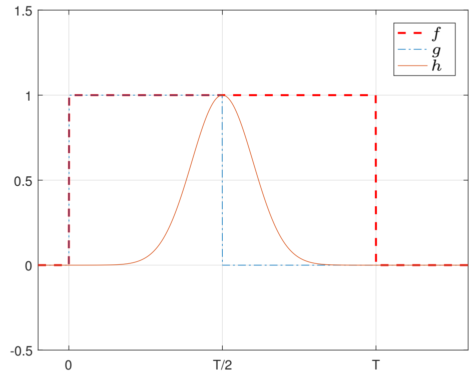

Fig. 2. Examples of pulse shapes

Some known code lines are RZ and NRZ which are abbreviations for return to zero and not return to zero, which relate to g and f correspondingly. Another division is uni-polar and bi-polar which refer to the symbols, $\{0, 1\}$ or $\{-1, 1\}$.

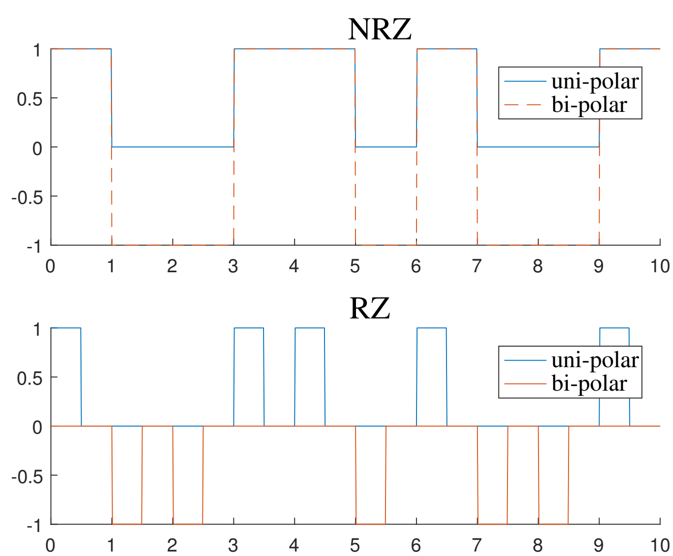

Fig. 3. Examples of code lines for the bit set (1,0,0,1,1,0,1,0,0,1)

II. WIRELESS CHANNEL MODEL AND BASIC DEFINITIONS.

Every communication system sends and receives information. Sending data is made in base station (Tx), while the receiving end is called terminal station (Rx). The direct line in between the base and the target stations is called *line of sight* (LOS), in this case we receive a noise-less version of the signal that have been sent:

$$y(t) = a \cdot s(t - \tau). \tag{1}$$

Most cases are not so simple, the signal hits and reflected from obstacles which causes attenuation and various delays, this is called multipath.

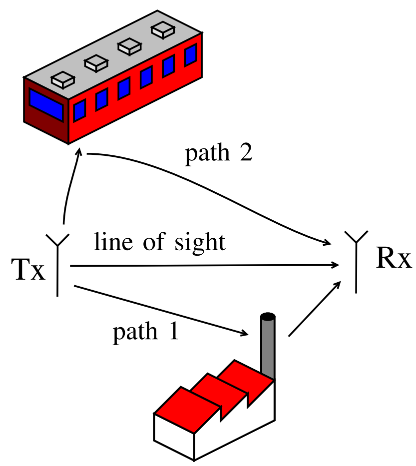

Fig. 4. Example of LOS and some reasonable paths.

The received signal for K different path-contributions will look like:

$$y(t) = \sum_{k=1}^{K} a_k s(t - \tau_k) \tag{2}$$

This is reflected in the system's impulse response:

$$h(t) = \sum_{k=1}^{K} a_k \delta(t - \tau_k) \tag{3}$$

$$H(f) = \sum_{k=1}^{K} a_k \mathrm{e}^{-j2\pi f \tau_k} \tag{4}$$

Delay spread is defined as the biggest difference between two delays

$$\Delta\tau = \max_i(\tau_i) - \min_i(\tau_i). \tag{5}$$

Intersymbol interference (ISI) is a form of distortion of a signal in which one symbol interferes with subsequent symbols. This happens when delay spread is bigger than period time $\Delta\tau > T$.

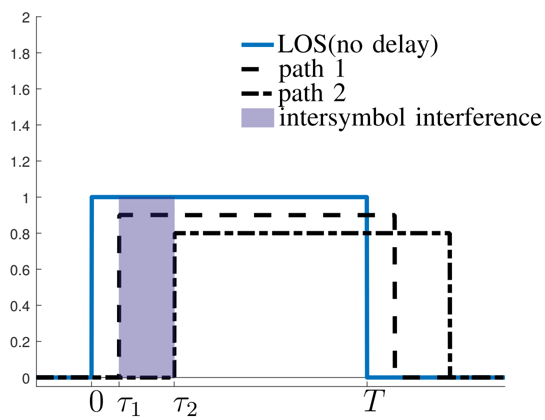

Fig. 5. An example of intersymbol interference on Real plane.

In frequency domain, ISI is is causing a frequency selective fading, i.e. every frequency has different amplitude. The desired signal is flat fading, which us equivalent to no fading

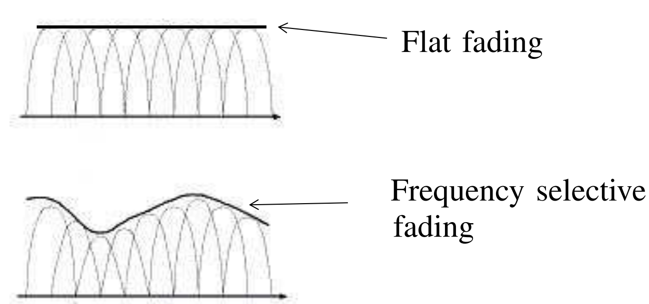

Fig. 6. Examples of frequency selective fading.

III. MULTI-CARRIER MODULATION

Communication world have managed to overcome ISI by using multi-carrier modulation (MCM). The general idea that lead to MCM is very simple, sending as much information as possible i.e. highest throughput. This require the bandwidth to be as big as possible.

$$B \triangleq f_{max} - f_{min} \tag{6}$$

$$B_{baseband} = f_{max}. \tag{7}$$

The last is true since $f_{min} \approx 0$. The main problem of this is that broad BW can cause severe ISI, or huge selective fading in frequency domain. Denote that $B = \frac{1}{T}$, so we get

$$\uparrow B \;\Rightarrow\; \downarrow T, \tag{8}$$

Delay spread is not affected by the band width i.e. constant, so we have

$$T \ll \Delta\tau. \tag{9}$$

This means that ISI have worsen. The solution to this trade-off is to work with multiple narrow bandwidths, hence dividing the original B to N narrow bands, each width is $\frac{B}{N}$. This is shown in the following equation that for each narrow band

$$\frac{B}{N} \;\Rightarrow\; T \cdot N, \tag{10}$$

Again $\Delta\tau$ is constant so he end up with

$$N \cdot T \gg \Delta\tau. \tag{11}$$

Which mean much less or no ISI at all. Slicing to narrow BW, or using MCM, solves the ISI problem. The idea is to pass the signal through S/P (serial to parallel) device and then multiply each symbol with different carrier f_i where

$$f_i = i \cdot \frac{B}{N} \qquad i = 0, 1, .., N-1 \tag{12}$$

The amount of parallel outputs N is in fact the factor that multiply the transfer rate, i.e. N times more symbols for the same time. This expresses in the following baseband signal

$$s(t) = \sum_{i=1}^{K} s_i \tag{13}$$
$$= \sum_{i=1}^{K} x_i \mathrm{e}^{j2\pi \frac{B}{N} it} \tag{14}$$
$$= \sum_{i=1}^{K} x_i \mathrm{e}^{j2\pi f_i t} \tag{15}$$

In order to differentiate the data from each narrow band, the receiver just need to multiply with correspond carrier frequency and then integrate it. Each narrow band is $\frac{B}{N} = \frac{1}{T} \cdot \frac{1}{N}$,this change the upper limit of the receiver's integrator to be $\tilde{T} = \frac{1}{B} = NT$.

$$\hat{x}_i(t) = \frac{1}{N}\frac{1}{T} \int_0^{NT} s(t) \mathrm{e}^{-j2\pi f_i t} dt \tag{16}$$
$$= \frac{1}{NT} \int_0^{NT} \left(\sum_k x_k \mathrm{e}^{j2\pi f_k t} \right) \mathrm{e}^{-j2\pi f_i t} dt \tag{17}$$
$$= \frac{1}{NT} \int_0^{NT} \left(\sum_k x_k(t) \mathrm{e}^{j2\pi \frac{kB}{N} t} \right) \mathrm{e}^{-j2\pi \frac{iB}{N} t} dt \tag{18}$$
$$= \frac{1}{NT} \sum_k \int_0^{NT} x_k \mathrm{e}^{j2\pi \frac{B}{N}(k-i)t} dt \tag{19}$$
$$= \begin{cases} 0 & i \neq k \\ x_i & i = k \end{cases} \tag{20}$$

So far this looks very promising but other problems arouses , each f_i require an accurate oscillator. Nowadays phones use large amount of carriers, such as $N = 1024$, which expresses in the need for too many oscillators in each device. This is not feasible due to the physical sizes of the components.

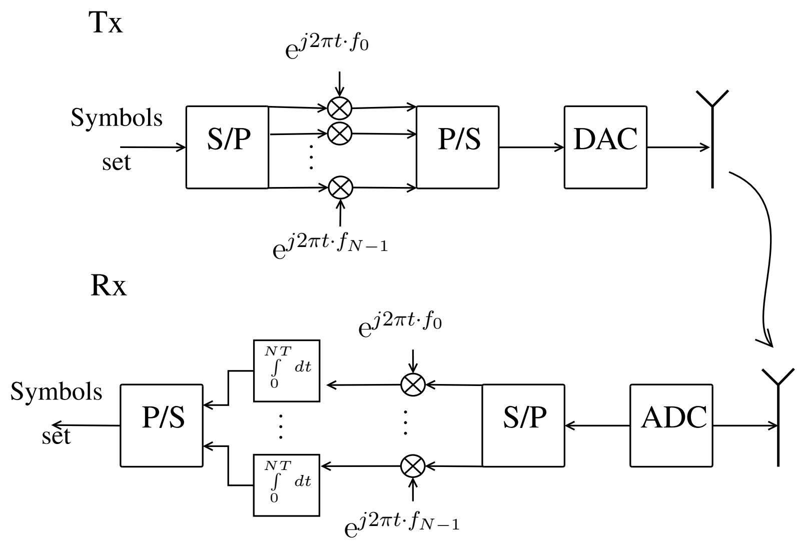

Fig. 7. Scheme of MCM transmitter and receiver

IV. OFDM

Orthogonal frequency-division multiplexing (OFDM) is a method of encoding digital data on multiple carrier frequencies. OFDM uses a digital multi-carrier modulation method, i.e. it deminish the need of oscillators. OFDM was introduced by Chang of Bell Labs in 1966[3]. The main advantage[2] of OFDM over single-carrier schemes is its ability to cope with severe channel conditions (for example, narrowband interference and frequency-selective fading due to multipath) without complex filters.

Reminder DFT\IDFT definition: for a discrete set $x[n]$ $, n = 0, 1, .., N-1$, we define:

$$X[k] = \text{DFT}\{x[n]\} = \sum_{n=0}^{N-1} x[n] \cdot \mathrm{e}^{-\frac{j2\pi kn}{N}} \tag{21}$$

$$x[n] = \text{IDFT}\{X[k]\} = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \cdot \mathrm{e}^{\frac{j2\pi kn}{N}} \tag{22}$$

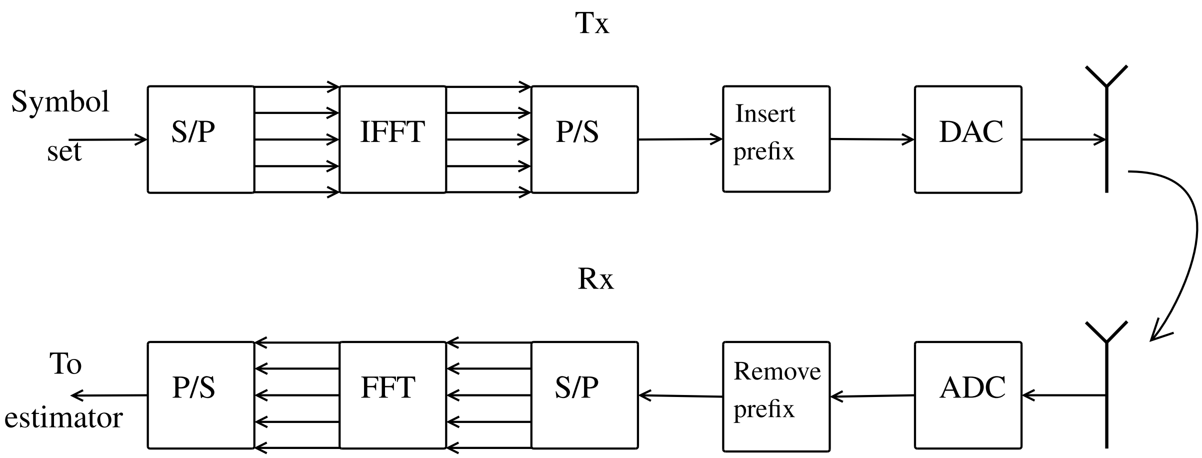

Fig. 8. Scheme of OFDM system.

A. Explanation I

Consider the MCM signal

$$x(t) = \frac{1}{N} \sum_k X_k \mathrm{e}^{\frac{2\pi k B}{N} t} \tag{23}$$

where X_k are the symbol of the k'th carrier. This reflects in a bandwidth size B, therefore Nyquist sampling rate will be $T = \frac{1}{B}$. Consider the sampling of the MCM signal with Nyquist sampling rate, we can rewrite x as:

$$\begin{aligned} x[n] &= x(nT) && (24)\\ &= \frac{1}{N} \sum_k X_k \mathrm{e}^{\frac{j2\pi k B}{N} nT} && (25)\\ &= \frac{1}{N} \sum_k X_k \mathrm{e}^{\frac{j2\pi k B}{N} \frac{n}{B}} && (26)\\ &= \frac{1}{N} \sum_k X_k \mathrm{e}^{\frac{j2\pi k n}{N}} && (27)\\ &= \mathrm{IFFT}\{X[k]\} && (28) \end{aligned}$$

Since we use IFFT in base station it is similar to treating the samples as if they were sampled on frequency domain. In order to simplify the problem and get a product problem which is easier to solve, we would like to have

$$y(n) = x(n) \cdot h(n) + w(n) \tag{29}$$

where $h(n)$ represent the channel, $w(n)$ stands for noise. But in fact there is

$$Y(k) = X(k) * H(k) + W(k). \tag{30}$$

One can suggest using the DFT for transforming the convolution to multiplication. This will not work straightforward because

$$\text{FFT}\{x \cdot y\} = X \circledast_N Y. \tag{31}$$

Therefore we insert prefixes to turn the convolution operation into cyclic convolution. Prefixes has more uses like decreasing ISI, improving BER , etc.

B. Explanation II

OFDM may be the explained using the matrix notation of DFT.

$$\text{DFT}\{x[n]\} = \underline{\underline{F}} \cdot \underline{x} = \underline{X} \tag{32}$$

$$\text{IDFT}\{X[k]\} = \frac{1}{N}\underline{\underline{F}}^H \cdot \underline{X} = \underline{x} \tag{33}$$

Where F, x are

$$F = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \mathrm{e}^{-\frac{j2\pi}{N}} & \mathrm{e}^{-\frac{j2\pi \cdot 2}{N}} & \dots & \mathrm{e}^{-\frac{j2\pi(N-1)}{N}} \\ \vdots & & \vdots & \ddots & \vdots \\ 1 & \mathrm{e}^{-\frac{j2\pi(N-1)}{N}} & \mathrm{e}^{-\frac{j2\pi \cdot 2(N-1)}{N}} & \dots & \mathrm{e}^{-\frac{j2\pi(n-1)^2}{N}} \end{bmatrix}, \quad x = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} \tag{34}$$

F is known as DFT matrix,an orthogonal matrix, where the element on the n'th row and m'th column is $F_{n,m}$,and $\underline{X}$ is the DFT vector of x.

$$F_{n,m} = \exp\left(-j2\pi \frac{n \cdot m}{N}\right) \quad n, m = 0, 1, .., N-1 \tag{35}$$

$$\frac{1}{N}\underline{\underline{F}}^H \underline{\underline{F}} = I_N \tag{36}$$

$\frac{1}{N}\underline{\underline{F}}^H \cdot \underline{X}$ makes a vector of

$$X[k] = \sum_n x[n]\mathrm{e}^{-\frac{j2\pi nk}{N}}. \tag{37}$$

As you can see using these matrix multiplications, each symbol is multiplied by the corresponding exponent, to form the transmitted signal.

C. Explanation III

Intuitive understanding the OFDM system may be explained with the following simplified scheme. So the relations between signals is as following:

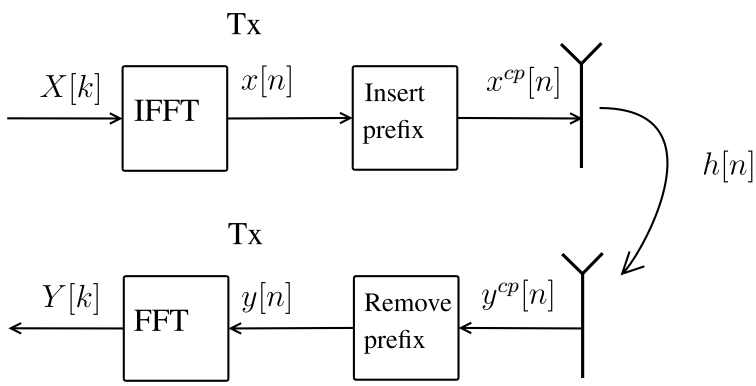

Fig. 9. Simplyfied OFDM scheme

$$x[n] = \text{IFFT}\{X[k]\} \tag{38}$$

$$y^{cp}[n] = x^{cp}[n] * h[n] + w[n] \tag{39}$$

$$= x[n] \circledast_N h[n] + w[n] \tag{40}$$

Note that the convolution turned to cyclic convolution due to inserting prefixes.

$$Y[k] = \text{FFT}\{y[n]\} \tag{41}$$

$$= \text{FFT}\{x[n] \circledast_N h[n] + w[n]\} \tag{42}$$

$$\overset{(a)}{=} X[k] \cdot H[k] + W[k] \tag{43}$$

Where (a) would not be possible without prefixes addition and removal. Difference between that simplified system and the complete one is mainly DAC/ADC which are pulse shaping and sampling.

V. MIMO DETECTION

Until this point, all signals were scalars which does not meet up to real world communication standards. Assuming we have t antennas at base station, and r antennas at terminal we can define the Multiple Input Multiple Output detection problem with the following equation:

$$Y = HX + W \tag{44}$$

Where $H \in \mathbb{C}^{r \times t}$ is a matrix that represent the channels between each of the antennas, $X \in \mathbb{C}^{t}$ is a vector of symbols from some constellation, i.g. 4-QAM, 16-QAM etc. and $W \in \mathbb{C}^{r}$ complex noise. The case that $t = r = 1$ is what is discussed in previous sections. Our goal is to have a ML algorithm whose input is H and Y and will produce the correct X, i.e. detect the symbol that had been sent. The assumption is that ML algorithm will solve this much faster. Taking in consideration that using OFDM and a new chip that can execute parallel computations, these can increase data transfer rate by big factors. This part will be further discussed in MIMO detection lecture.

REFERENCES

[1] Doron Ezri. *MIMO-OFDM LECTURE*
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[2] *The history of orthogonal frequency-division multiplexing*.
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